Property

1. $a^m \cdot a^n = a^{m+n}$

2. $(a^m)^n = a^{mn}$

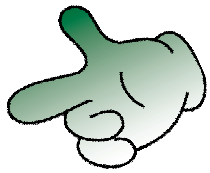
3. $(ab)^m = a^m b^m$

4. $a^{-m} = \frac{1}{a^m}, a \neq 0$

5. $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$

6. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$

$$\frac{x^8}{x^{-2}} = x^{10}$$



Know the basic properties of exponents.

You need to be able to find roots of powers of common bases.
(Be familar with those on the cheat-sheet that I gave you.)

$$125^{\frac{2}{3}}$$

$$\left(\sqrt[3]{125}\right)^2 = 5^2 = 25$$

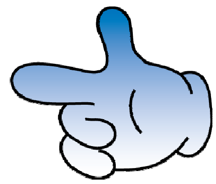
$$(-32)^{\frac{3}{5}}$$

$$\left(\sqrt[5]{-32}\right)^3 = (-2)^3 = -8$$

$$9^{\frac{5}{2}} = \left(\sqrt[2]{9}\right)^5 = 3^5$$

$$\begin{array}{r} 81 \\ \times\ 3 \\ \hline 243 \end{array}$$

$$\sqrt[3]{64} = 64^{\frac{1}{3}}$$

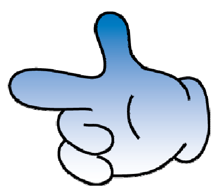


You need to know how to convert back and forth between *radical* form and *rational exponent* form.

$$\left(\sqrt[5]{11}\right)^4 = 11^{\frac{4}{5}} \quad \text{or} \quad \sqrt[5]{11^4}$$

$$8^{\frac{4}{3}} = \sqrt[3]{8^4} \quad \text{or} \quad \left(\sqrt[3]{8}\right)^4 = (2)^4 = 16$$

Solve:



You need to know how to solve an equation by isolating the base, then extracting the appropriate root.

$$\frac{6x^3}{6} = \frac{384}{6}$$

$$\sqrt[3]{x^3} = \sqrt[3]{64}$$

$$x = 4$$

$$\sqrt[5]{(x - 8)^5} = \sqrt[5]{100}$$

$100^{\frac{1}{5}}$

100 ^ (1/5)

$$x - 8 = 2.5$$

$$x = 10.5$$

$$(x-8)^5 = 100$$

$+8$

KEY CONCEPT — *For Your Notebook*

Operations on Functions

Let f and g be any two functions. A new function h can be defined by performing any of the four basic operations on f and g.

Operation	Definition	Example: $f(x) = 5x$, $g(x) = x + 2$
Addition	$h(x) = f(x) + g(x)$	$h(x) = 5x + (x + 2) = 6x + 2$
Subtraction	$h(x) = f(x) - g(x)$	$h(x) = 5x - (x + 2) = 4x - 2$
Multiplication	$h(x) = f(x) \cdot g(x)$	$h(x) = 5x(x + 2) = 5x^2 + 10x$
Division	$h(x) = \frac{f(x)}{g(x)}$	$h(x) = \frac{5x}{x + 2}$

The domain of h consists of the x-values that are in the domains of both f and g. Additionally, the domain of the quotient does not include x-values for which $g(x) = 0$.

$f(x) = 8x^3 - 4x + 7$ D=Real

x = Real

$f(x) = \sqrt{x}$ Domain $x \geq 0$

$f(x) = \frac{x^2+3}{x-5}$ Domain: x=Real, $x \neq 5$

$f(x) = \sqrt[3]{x}$ Domain: x=Real

$\sqrt[3]{-8} = -2$

$f(x) = x^{\frac{1}{2}}$ D: $x \geq 0$

KEY CONCEPT *For Your Notebook*

Operations on Functions

Let f and g be any two functions. A new function h can be defined by performing any of the four basic operations on f and g.

Operation	Definition	Example: $f(x) = 5x$, $g(x) = x + 2$
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Division	$h(x) = \dfrac{f(x)}{g(x)}$	$h(x) = \dfrac{5x}{x + 2}$

The domain of h consists of the x-values that are in the domains of both f and g. Additionally, the domain of the quotient does not include x-values for which $g(x) = 0$.

$8x^3 - 2x^2 + 7$ $\quad$ $\mathbb{R}$

$\sqrt{x} + 8$ $\quad$ $x \geq 0$

$\dfrac{x^2 - 7}{x + 5}$ $\quad$ $\mathbb{R}$; $x \neq -5$

Let $f(x) = 5x^{1/3}$ and $g(x) = -11x^{1/3}$

a. find $f(x) + g(x)$ $5x^{\frac{1}{3}} + -11x^{\frac{1}{3}} = -6x^{\frac{1}{3}}$

b. find $f(x) - g(x)$ $5x^{\frac{1}{3}} - -11x^{\frac{1}{3}} = 16x^{\frac{1}{3}}$

Let f(x) = 8x and g(x) = $2x^{5/6}$

Find f(x) * g(x) $= 8x^{1} \cdot 2x^{\frac{5}{6}} = 16x^{\frac{11}{6}}$

$1 + \frac{5}{6} = 1\frac{5}{6} = \frac{11}{6}$

Find $\frac{f(x)}{g(x)} = \frac{8x^{1}}{2x^{\frac{5}{6}}} = 4x^{\frac{1}{6}}$ $\left[4\sqrt[6]{x}\right]$

$x \geq 0$

composition of functions

f(g(x))

Perform function "g" on x, to find g(x). This becomes the input value that you input into function "f."

$f(x) = 3x - 4$ and $g(x) = x^2 - 1$

This answer is a <u>number</u>.

What is the value of f(g(-3))?

$g(-3) = (-3)^2 - 1 = 9 - 1 = 8$

$f(8) = 3(8) - 4 = 20$

Write a simplified expression for f(g(x)).

These answers are <u>expressions.</u>

$3(x^2 - 1) - 4$

$3x^2 - 3 - 4$

$f(g(x)) = 3x^2 - 7$

Write a simplified expression for g(f(x)).

$(3x-4)^2 - 1$

$\cancel{9x^2 - 16}$

$(3x-4)(3x-4)$

$9x^2 - 12x - 12x + 16$

$9x^2 - 24x + 16$

$9x^2 - 24x + 16 - 1$

$g(f(x)) = 9x^2 - 24x + 15$

composition of functions

$f(g(x))$

Perform function "g" on x, to find $g(x)$. This becomes the input value that you input into function "f."

$f(x) = 3x - 4$ and $g(x) = x^2 - 1$

What is the value of $f(g(-3))$

Review: Composition of Functions

$f(x) = 2x + 1$ $\quad$ $g(x) = 3x - 5$

Find $f(g(x)) = 2(3x-5)+1$

$= 6x-10+1$

$= 6x-9$

Find $g(f(x)) = 3(2x+1)-5$

$= 6x+3-5 = 6x-2$

READING
The symbol -1 in f^{-1} is not to be interpreted as an exponent. In other words, $f^{-1}(x) \neq \frac{1}{f(x)}$.

KEY CONCEPT *For Your Notebook*

Inverse Functions

Functions f and g are inverses of each other provided:

$$f(g(x)) = x \qquad \text{and} \qquad g(f(x)) = x$$

The function g is denoted by f^{-1}, read as "f inverse."

Find and verify the inverse:

$$y = 4x + 2$$

1. Switch x and y
2. Solve for y.

$$x = 4y + 2$$

$$\frac{x}{4} - \frac{2}{4} = \frac{4y}{4}$$

$$\frac{x}{4} - \frac{1}{2} = y$$

Verify:

Show that $f(f^{-1}(x)) = x$.

$$4\left(\frac{x}{4} - \frac{1}{2}\right) + 2$$

$$x - 2 + 2$$

$$x$$

Show that $f^{-1}(f(x)) = x$.

$$\frac{(4x+2)}{4} - \frac{1}{2}$$

$$x + \frac{1}{2} - \frac{1}{2} = x$$

Find and verify the inverse: $y = -\frac{2}{3}x + 2$

$$x = -\frac{2}{3}y + 2$$

$$-\frac{3}{2}(x-2) = -\frac{2}{3}y \cdot -\frac{3}{2}$$

$$-\frac{3}{2}x + 3 = y$$

$$-\frac{2}{3}\left(-\frac{3}{2}x + 3\right) + 2$$

$$x - 2 + 2$$

$$x$$

$$-\frac{3}{2}\left(-\frac{2}{3}x + 2\right) + 3$$

$$x - 3 + 3 = x$$

verify that $f(x) = 4x + 2$ and $f^{-1}(x) = \frac{1}{4}x - \frac{1}{2}$ are inverse functions.

$$f(f^{-1}(x)) = 4\left(\frac{1}{4}x - \frac{1}{2}\right) + 2$$

$$x - 2 + 2$$

$$x$$

$$f^{-1}(f(x) = \frac{1}{4}(4x + 2) - \frac{1}{2}$$

$$x + \frac{1}{2} - \frac{1}{2}$$

$$x$$

verifying inverses

$f(x) = 5x^2 - 2$, $x \geq 0$; $g(x) = \left(\frac{x+2}{5}\right)^{1/2}$

$f(x) = 5x^2 - 2$
$g(x) = \left(\frac{x+2}{5}\right)^{\frac{1}{2}}$

Show that $f(g(x)) = x$ and $g(f(x)) = x$

$$f(g(x)) = 5\left(\left(\frac{x+2}{5}\right)^{\frac{1}{2}}\right)^2 - 2$$

$$= 5\left(\frac{x+2}{5}\right) - 2$$

$$= x + 2 - 2 = x$$

$$g(f(x)) = \left(\frac{(5x^2-2)+2}{5}\right)^{\frac{1}{2}}$$

$$= \left(\frac{5x^2}{5}\right)^{\frac{1}{2}} = \left(x^2\right)^{\frac{1}{2}} = x$$

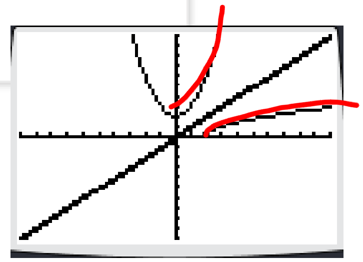
KEY CONCEPT *For Your Notebook*

Inverse Functions

Functions f and g are inverses of each other provided:

$$f(g(x)) = x \qquad \text{and} \qquad g(f(x)) = x$$

The function g is denoted by f^{-1}, read as "f inverse."



$f(x) = x^2 + 2$

$x \geq 0$

$y = x^2 + 2$

$x = y^2 + 2$

$\sqrt{x-2} = \sqrt{y^2}$

$y = \sqrt{x-2}$

$f(x) = -x^3 + 4$

$y = -x^3 + 4$

$x = -y^3 + 4$

$x - 4 = -y^3$

$\sqrt[3]{-x+4} = \sqrt[3]{y^3}$

$y = \sqrt[3]{-x+4}$

inverses of functions

If in terms of f(x)= or y =

Switch x and y.

Solve for y.

$f(x) = 2x^2 - 3$

$y = 2x^2 - 3$

$x = 2y^2 - 3$

$+3 \qquad +3$

$\frac{x+3}{2} = \frac{2y^2}{2}$

$\sqrt{\frac{x+3}{2}} = \sqrt{y^2}$

$\sqrt{\frac{x+3}{2}} = y$

If it is in a context, such as

Do not switch.

Solve for the other variable.

$F = \frac{9}{5}C + 32$

$-32 \qquad -32$

$\frac{5}{9}(F - 32) = \frac{9}{5}C \cdot \frac{5}{9}$

$\frac{5}{9}(F - 32) = C$

P.452

KEY CONCEPT *For Your Notebook*

Solving Radical Equations

To solve a radical equation, follow these steps:

STEP 1 **Isolate** the radical on one side of the equation, if necessary.

STEP 2 **Raise** each side of the equation to the same power to eliminate the radical and obtain a linear, quadratic, or other polynomial equation.

STEP 3 **Solve** the polynomial equation using techniques you learned in previous chapters. Check your solution.

$\sqrt[3]{x} - 9 = -1$

$+9 \quad +9$

$(\sqrt[3]{x})^3 = (8)^3$

$x = 512$

*do not subtract 25 first!

$(\sqrt[3]{x+25})^3 = 4^3$

$\sqrt[3]{x+25} = 4$

$x + 25 = 64$

$-25 \quad -25$

$x = 39$

$\frac{\cancel{2}\sqrt[3]{x-3}}{\cancel{2}} = \frac{4}{2}$

$(\sqrt[3]{x-3})^3 = 2^3$

$x - 3 = 8$

$x = 11$

$\frac{7x^{3/5}}{7} = \frac{56}{7}$

$\left(x^{\frac{3}{5}}\right)^{\frac{5}{3}} = 8^{\frac{5}{3}}$

$x = 32$

$(\sqrt[3]{8})^5 = 2^5$

$= 32$

$(x - 4)^{2/3} - 9 = 16$

$+9 \quad +9$

$\left((x-4)^{\frac{2}{3}}\right)^{\frac{3}{2}} = (25)^{\frac{3}{2}}$

$(\sqrt{25})^3 = 5^3$

$= 125$

$x - 4 = 125$

$x = 129$

KEY CONCEPT *For Your Notebook*

Solving Radical Equations

To solve a radical equation, follow these steps:

STEP 1 **Isolate** the radical on one side of the equation, if necessary.

STEP 2 **Raise** each side of the equation to the same power to eliminate the radical and obtain a linear, quadratic, or other polynomial equation.

STEP 3 **Solve** the polynomial equation using techniques you learned in previous chapters. Check your solution.

$\sqrt[3]{x} - 9 = -1$

$+9 \quad +9$

$\left[\sqrt[3]{x}\right]^3 = (8)^3 \qquad \left(x^{\frac{1}{3}}\right)^{\frac{3}{1}}$

$x = 512 \qquad \sqrt[3]{512} - 9 = -1$

$8 - 9 = -1$ ✓

$\left[\sqrt[3]{x+25}\right]^3 = (4)^3$

$x + 25 = 64 \qquad \sqrt[3]{39+25} = 4$

$x = 39 \qquad \sqrt[3]{64} = 4$ ✓

$\dfrac{2\cdot\sqrt[3]{x-3}}{2} = \dfrac{4}{2}$

$\left(\sqrt[3]{x-3}\right)^3 = 2^3 \qquad x - 3 = 8$

$x = 11$

$2\sqrt[3]{11-3} = 4$

$2\cdot 2 = 4$ ✓

$\dfrac{7\cdot x^{3/5}}{7} = \dfrac{56}{7}$

$\left(x^{3/5}\right)^{\frac{5}{3}} = (8)^{\frac{5}{3}} \qquad 7\sqrt[5]{x^3}$

$x = 32 \qquad 7(32)^{3/5} = 56$

$(x - 4)^{2/3} - 9 = 16 \qquad 7(2^3) = 56$ ✓

$\left[(x-4)^{2/3}\right]^{\frac{3}{2}} = (25)^{\frac{3}{2}}$

$x - 4 = \pm 125 \qquad 25^{\frac{3}{2}} = \left(\sqrt{25}\right)^3$

$x = 129$ OR -121

$125 = (5)^3$

$-125 = (-5)^3$

Solving Radical Equations:

$$\left(\sqrt{2x+3}\right)^2 = 7^2$$

$$2x+3=49$$

$$2x=46$$

$$x=23$$

$$-5\sqrt{x+1}+12=2$$

$$-12 \quad -12$$

$$\frac{-5\sqrt{x+1}}{-5} = \frac{-10}{-5}$$

$$\left(\sqrt{x+1}\right)^2 = 2^2$$

$$x+1=4 \qquad x=3$$

$$x^2 = \left(\sqrt{4x-3}\right)^2 \qquad x=\sqrt{4x-3}$$

$$x^2=4x-3$$

$$x^2-4x+3=0$$

$$(x-3)(x-1)=0$$

$$x=3,1$$

$$\left(\sqrt{x^2+4}\right)^2 = (x+5)^2 \qquad \sqrt{x^2+4}=x+5$$

FOIL

$$\cancel{x^2}+4=\cancel{x^2}+10x+25$$

$$4=10x+25$$

$$-25 \quad -25$$

$$\frac{-21}{10}=\frac{10x}{10}$$

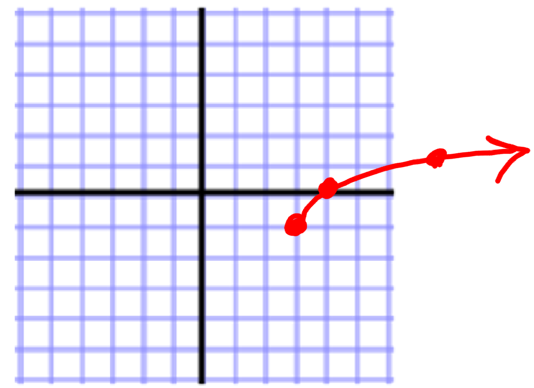
$$-\frac{21}{10}=x$$

Graphs

Square roots (and other even roots)

$y = \sqrt{x-3} - 1$

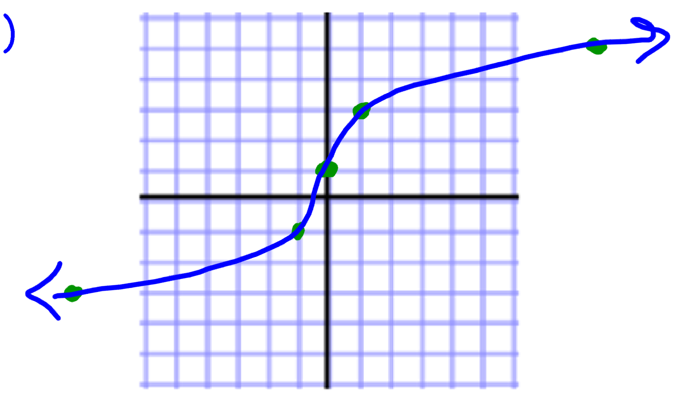
x	y
3	-1
4	0
7	1



Cube roots (and other odd roots)

$y = 2\sqrt[3]{x} + 1$

x	y
0	1
1	3
8	5
-1	-1
-8	-3



Name________________________________ Algebra 2 Take-Home Quiz (6.3-6.4)

Let $f(x) = 5x^2$ and $g(x) = 3x^2$. Perform the indicated operation and state the domain.

1. $f(x) + g(x)$

$5x^2 + 3x^2$

$8x^2$ 1

D: Real $\frac{1}{2}$

2. $f(x) - g(x)$

$5x^2 - 3x^2$

$2x^2$

D: Real

3. $f(x) \cdot g(x)$

$(5x^2)(3x^2)$

$15x^4$

D: Real

4. $\frac{f(x)}{g(x)}$

$\frac{5\cancel{x^2}}{3\cancel{x^2}}$

$\frac{5}{3}$ D: Real

$x \neq 0$

5. $f(g(x))$

$5(3x^2)^2$

$5(3x^2)(3x^2)$

$45x^4$

Real

5. $g(f(x))$

$3(5x^2)^2$

$3(5x^2)(5x^2)$

$75x^4$

Real

Verify that f and g are inverse functions.

7. $f(x) = x - 7$; $g(x) = x + 7$

$f(g(x)) = (x+7) - 7$

x

$g(f(x)) = (x-7) + 7$

$= x$

8. $f(x) = 6x^3$; $g(x) = \sqrt[3]{\frac{x}{6}}$

$f(g(x)) = 6\left(\sqrt[3]{\frac{x}{6}}\right)^3$

$\cancel{6}\left(\frac{x}{\cancel{6}}\right)$

$= x$

$g(f(x)) = \sqrt[3]{\frac{(\cancel{6}x^3)}{\cancel{6}}}$

$= \sqrt[3]{x^3}$

$= x$

Find the inverse of the function.

9. $f(x) = 3x + 10$

$y = 3x + 10$

$x = 3y + 10$

$-10 \qquad -10$

$\frac{x-10}{3} = \frac{3y}{3}$

$\frac{x-10}{3} = y$

10. $f(x) = -\frac{3}{8}x^7$

$y = -\frac{3}{8}x^7$

$\left(\frac{-8}{3}\right)x = \cancel{-\frac{3}{8}} \cdot y^7 \cancel{\left(\frac{-8}{3}\right)}$

$\sqrt[7]{-\frac{8}{3}x} = \sqrt[7]{y^7}$

$\sqrt[7]{-\frac{8}{3}x} = y$

11. $f(x) = x^2 - 9$, $x \geq -9$

$y = x^2 - 9$

$x = y^2 - 9$

$\sqrt{x+9} = \sqrt{y^2}$

$\sqrt{x+9} = y$

12. The cost (in dollars) of *g* gallons of gasoline can be modeled by $C(g) = 3.15g$. The amount of gasoline used by an SUV can be modeled by $g(d) = 0.025d^{1.24}$ where d is the distance (in miles). Find $C(g(d))$. Evaluate $C(g(500))$. What does $C(g(500))$ represent?

$C(g(d)) = 3.15(0.025d^{1.24})$

$g(500) = 0.025(500)^{1.24}$

$g = 55.55$ (gallons)

$C(55.55) = 3.15(55.55)$

$= \$174.97$

Cost to drive 500 miles

Exponential Growth and Decay

$$y = a * b^{x}$$

a = starting value (initial value) or value when x = 0

b = growth (or decay) factor   b > 1 grows
b < 1 decays

Increase: b = 1 + % increase

Decrease: b = 1 - % decrease

3 % increase: $b = 1 + .03 = 1.03$

3% decrease: $b = 1 - .03 = 0.97$

80% increase: $b = 1 + .8 = 1.8$

80% decrease: $b = 1 - .8 = 0.2$

100% increase: $b = 1 + 1 = 2$

KEY CONCEPT *For Your Notebook*

Compound Interest

Consider an initial principal P deposited in an account that pays interest at an annual rate r (expressed as a decimal), compounded n times per year. The amount A in the account after t years is given by this equation:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Calculate the value of $800 invested for 5 years. Interest is compounded monthly at 4%.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 800\left(1 + \frac{.04}{12}\right)^{12 \cdot 5}$$

$976.80

Use your calculator to make a graph, but set the window so that it shows the data for the relevant domain and range.

41. ★ **EXTENDED RESPONSE** In 2000, the average price of a football ticket for a Minnesota Vikings game was \$48.28. During the next 4 years, the price increased an average of 6% each year.

$6\% \rightarrow .06 + 1$

a. Write a model giving the average price p (in dollars) of a ticket t years after 2000.

b. Graph the model. Estimate the year when the average price of a ticket was about \$60.

c. *Explain* how you can use the graph of $p(t)$ to determine the minimum and maximum p-values over the domain for which the function gives meaningful results.

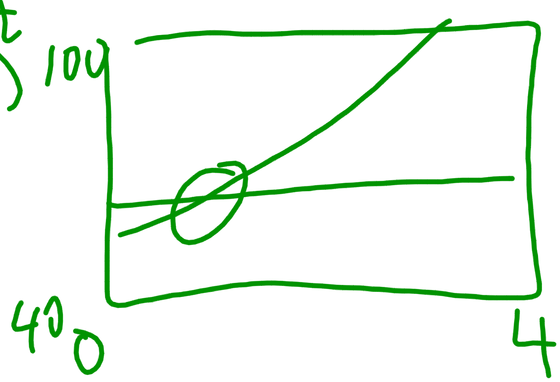
$y = a \cdot b^x$ growth factor

start value / value when $x=0$

$y = 48.28(1.06)^x$

$p = 48.28(1.06)^t$

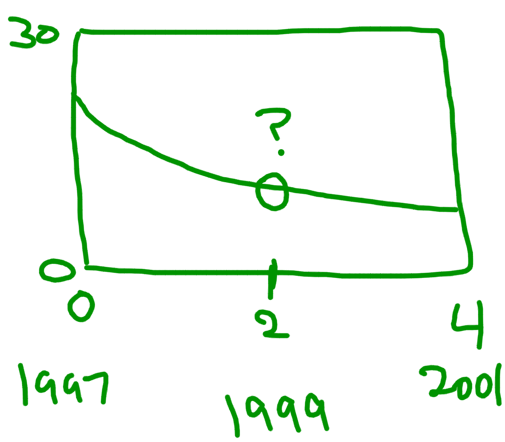
$Y_2 = 60$



TV SALES From 1997 to 2001, the number n (in millions) of black-and-white TVs sold in the United States can be modeled by $n = 26.8(0.85)^t$ where t is the number of years since 1997. Identify the decay factor and the percent decrease. Graph the model and state the domain and range. Estimate the number of black-and-white TVs sold in 1999. *(p. 486)*

15%

15% decrease



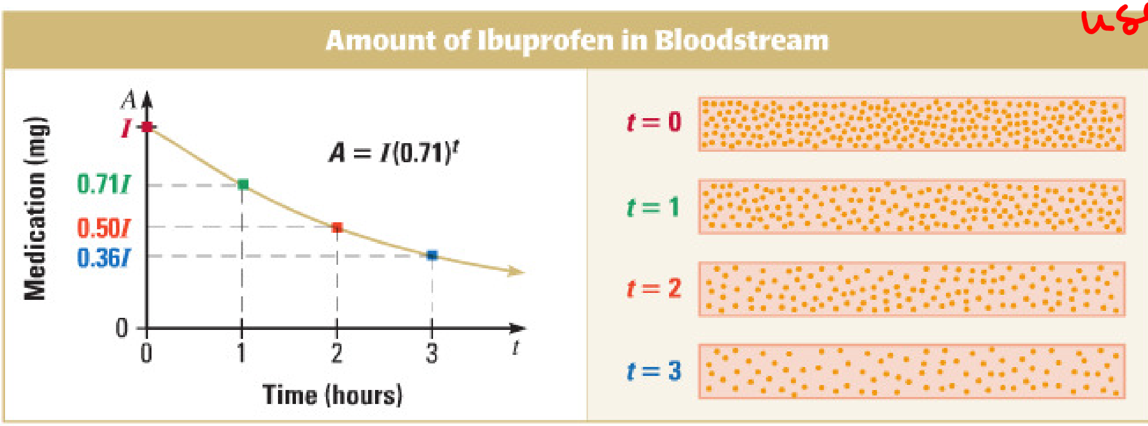
Set your window so that you see the graph for the relevant domain and range.

decay factor 0.85

% decrease 15%

30. MEDICINE When a person takes a dosage of I milligrams of ibuprofen, the amount A (in milligrams) of medication remaining in the person's bloodstream after t hours can be modeled by the equation $A = I(0.71)^t$.

71% remains
29% used



Find the amount of ibuprofen remaining in a person's bloodstream for the given dosage and elapsed time since the medication was taken.

a. Dosage: 200 mg
Time: 1.5 hours

b. Dosage: 325 mg
Time: 3.5 hours

c. Dosage: 400 mg
Time: 5 hours

$A = I(0.71)^t$

$= 200(0.71)^{1.5} = 119.65$ mg

$$\left(1 + \frac{1}{n}\right)^n$$

Natural Base e

The history of mathematics is marked by the discovery of special numbers such as π and i. Another special number is denoted by the letter e. The number is called the **natural base *e*** or the *Euler number* after its discoverer, Leonhard Euler (1707–1783). The expression $\left(1 + \frac{1}{n}\right)^n$ approaches e as n increases.

n	10^1	10^2	10^3	10^4	10^5	10^6
$\left(1 + \frac{1}{n}\right)^n$	2.59374	2.70481	2.71692	2.71815	2.71827	2.71828

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

r = 100%
t = 1 year

$$A = 1000\left(1 + \frac{1}{n}\right)^n$$

KEY CONCEPT *For Your Notebook*

The Natural Base *e*

The natural base e is irrational. It is defined as follows:

As n approaches $+\infty$, $\left(1 + \frac{1}{n}\right)^n$ approaches $e \approx 2.718281828$.

Logarithm of y with base b.

answer

exponent

$\log_b y = x$ Read as "log base b of y."

base

$\log_2 8 = 3$ $2^3 = 8$

$\log_4 1 = 0$ $4^0 = 1$

$\log_4 1/4 = -1$ $4^{-1} = \frac{1}{4}$

$\log_3 81 = 4$ $3^4 = 81$

$\cancel{\log_2 x} = 5$

$\log_2$

so.... $y = b^x$ and $y = \log_b x$ are inverse functions

Two rules:

$$b^{\log_b x} = x \qquad\qquad \log_b b^x = x$$

Simplify:

a. $10^{\log_{10} 4} = 4$

b. $8^{\log_8 x} = x$

c. $\log_7 7^{-3x} = -3x$

d. $\log_2 16^x = \log_2 (2^4)^x = \log_2 2^{4x} = 4x$

e. $\log_3 27^x = \log_3 (3^3)^x = \log_3 3^{3x} = 3x$

$e^{2x} = 16$

$\ln e^{2x} = \ln 16$

Review of exponential rules

$$e^a e^b = e^{a+b}$$

$$\frac{e^a}{e^b} = e^{a-b}$$

$$\left(e^a\right)^b = e^{ab}$$

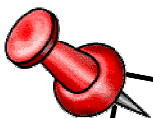
$$\ln e^1 = 1$$

Example: solve for x

$$\ln x = 3$$

$$e^{\ln x} = e^3$$

$$x = e^3$$



Review of LOG and LN rules

*The following rules also work for ***ln***

$$\log_b a = c \rightarrow b^c = a$$

$$\log_b (m \times n) = \log_b m + \log_b n$$

$$\log_b \left(\frac{m}{n} \right) = \log_b m - \log_b n$$

$$\log_b m^n = n \log_b m$$

Solving Exponential and Logarithmic Equations

Solving an Exponential Equation	Solving a Logarithmic Equation
If each side can be written using the same base, equate exponents. $3^{x+1} = 9^x$ $3^{x+1} = (3^2)^x$ $x + 1 = 2x$ $1 = x$	If the equation has the form $\log_b x = \log_b y$, use the fact that $x = y$. $\log_2 (4x - 2) = \log_2 3x$ $4x - 2 = 3x$ $x = 2$
If each side cannot be written using the same base, take a logarithm of each side. $6^x = 15$ $\log_6 6^x = \log_6 15$ $x = \frac{\log 15}{\log 6} \approx 1.511$	If a logarithm is set equal to a constant, exponentiate each side. $\log_5 (x + 1) = 2$ $x + 1 = 5^2$ $x = 24$

$$\cancel{5^{\log_5}}{}^{(x+1)} = 5^2$$

Let b, m, and n be positive numbers such that $b \neq 1$.

Product Property	$\log_b mn = \log_b m + \log_b n$
Quotient Property	$\log_b \frac{m}{n} = \log_b m - \log_b n$
Power Property	$\log_b m^n = n \log_b m$

Product Property:

Expand: $\log 15x = \log 15 + \log x$ $\qquad \log_3 2y$

Condense: $\log_3 4 + \log_3 y$ $\qquad \ln 2 + \ln x$

Quotient Property:

Expand: $\ln \frac{12}{5}$ $\qquad \log_5 \frac{x}{3}$

Condense: $\log_3 4 - \log_3 y$ $\qquad \ln x - \ln 3$

Expand: $\log_4 \frac{3}{2x}$

Condense: $\log_5 3 + \log_5 x - \log_5 2$

$\log_5\left(\frac{3x}{2}\right)$

Power Property:

Expand: $\log x^4$ $\qquad \log_2 m^5 = 5\log_2 m$

$\log_3 2x^4$ $\qquad \ln 8x^3$

$\ln 8 + \ln x^3$

$\ln 8 + 3 \ln x$

$\log_3 \sqrt{y} = \log_3 (y)^{\frac{1}{2}}$

$= \frac{1}{2} \log_3 y$

Condense:

$\ln 12 - \ln 4$

$\ln\left(\frac{12}{4}\right) = \ln 3$

$5 \log x - 4 \log y$

$\log x^5 - \log y^4$

$\log\left(\frac{x^5}{y^4}\right)$

$6 \ln 2 - 4 \ln y$

$\frac{\ln 2^6}{\ln y^4} = \frac{\ln 64}{\ln y^4}$

KEY CONCEPT *For Your Notebook*

Change-of-Base Formula

If a, b, and c are positive numbers with $b \neq 1$ and $c \neq 1$, then:

$$\log_c a = \frac{\log_b a}{\log_b c}$$

In particular, $\log_c a = \frac{\log a}{\log c}$ and $\log_c a = \frac{\ln a}{\ln c}$.

Example: $\log_5 12 = \frac{\log 12}{\log 5}$ OR $\frac{\ln 12}{\ln 5}$

Use the change-of-base formula to evaluate the logarithm.

7. $\log_5 8$ **8.** $\log_8 14$ **9.** $\log_{26} 9$ **10.** $\log_{12} 30$

$\frac{\log 8}{\log 5}$ $5^{?} = 8$

Solve: $7^{9x} = 15$

click to reveal

$7^{9x} = 15$

We need to turn this 15 into a power of 7.

Here's how: $\log_7 15$

$\log_7 7^{9x} = \log_7 15$ Take log of each side

$9x = \log_7 15$ Simplify

$9x = \dfrac{\log 15}{\log 7} = 1.392$ Use change-of-base

$x = 0.155$

click to reveal

$4e^{-0.3x} - 7 = 13$

Isolate the variable

$4e^{-0.3x} = 20$

$e^{-0.3x} = 5$

We need to turn this 5 into a power of e.

Here's how: ln 5

$\ln e^{-0.3x} = \ln 5$ Take log of each side

$-0.3x = 1.609$ Simplify

$x = -5.365$ Solve

click to reveal

Exponentiating to solve log equations

$\log_4 (5x - 1) = 3 \longrightarrow \log_4 (65-1) \stackrel{?}{=} 3$

$\log_4 64 = 3$ ✓

$4^{\log_4 (5x - 1)} = 4^3$ Exponentiate each side

$5x - 1 = 64$ Simplify

$5x = 65$ Solve

$x = 13$

Check to make sure your answer checks in the original equation !

Solve:

click to reveal

$$\ln (7x - 4) = \ln (2x + 11)$$

$$e^{\ln (7x - 4)} = e^{\ln (2x + 11)}$$ Exponentiate

$$7x - 4 = 2x + 11$$ Simplify

$$5x = 15$$ Solve

$$x = 3$$

Check to make sure your answer checks in the original equation !

Solve:

$$\log_2 (x - 6) = 5$$

$$2^{\log_2 (x - 6)} = 2^5$$ Exponentiate

$$x - 6 = 32$$ Simplify

$$x = 38$$ Solve

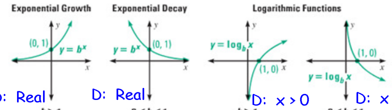
Check to make sure your answer checks in the original equation !

Chapter 7 Algebra 2 Study Guide

Graphing Exponential and Logarithmic Functions

Parent functions for exponential functions have the form $y = b^x$. Parent functions for logarithmic functions have the form $y = \log_b x$.

Exponential Growth | **Exponential Decay** | **Logarithmic Functions**



D: Real | D: Real | D: $x > 0$ | D: $x > 0$

$b > 1$ | $0 < b < 1$ | $b > 1$ | $0 < b < 1$

R: $y > 0$ | R: $y > 0$ | R: Real | R: Real

Always on the parent graph: (0 , 1) and (1 , b)

Always on the parent graph: (1 , 0) and (b , 1)

Exponential: $y = b^{(x-h)} + k$

horizontal asymptote at k

R: $y > k$

Horizontal shift: h
Vertical shift: k

Logarithmic: $y = \log_b (x - h) + k$

vertical asymptote at h

D: $x > h$

Compound Interest

Consider an initial principal P deposited in an account that pays interest at an annual rate r (expressed as a decimal), compounded n times per year. The amount A in the account after t years is given by this equation:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Continuously Compounded Interest

When interest is compounded *continuously*, the amount A in an account after t years is given by the formula

$$A = Pe^{rt}$$

where P is the principal and r is the annual interest rate expressed as a decimal.

Definition of Logarithm with Base *b*

Let b and y be positive numbers with $b \neq 1$. The **logarithm of y with base b** is denoted by $\log_b y$ and is defined as follows:

$$\log_b y = x \quad \text{if and only if} \quad b^x = y$$

The expression $\log_b y$ is read as "log base b of y."

Examples: $\log_3 81 = 4$ | $5^4 = 625$ | $\ln 6 = 1.792$ (e) | $\log 1000 = 3$

$3^4 = 81$ | $\log_5 625 = 4$ | $e^{1.792} = 6$ | $10^3 = 1000$

Properties of Logarithms

Let b, m, and n be positive numbers such that $b \neq 1$.

Product Property $\log_b mn = \log_b m + \log_b n$

Quotient Property $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power Property $\log_b m^n = n \log_b m$

Expand: $\log 8x^3$

$\log 8 + \log x^3$

$\log 8 + 3 \log x$

Condense: $5 \log_4 5 + 2 \log_4 x$

$\log_4 5^5 + \log_4 x^2$

$\log_4 5^5 \cdot x^2$

$\log_4 3125x^2$

$2^x = 15$

$\cancel{\log_2 2}^x = \log_2 15$

$x = \frac{\log 15}{\log 2} = 3.9$

$1.1436 \to 1.1$

$1.0827 \to 1.1$

$\frac{6 \log_4 3x}{6} = \frac{12}{6}$

$\log_4 3x = 2$

$\cancel{4^{\log_4}} 3x = 4^2$

$3x = 16$

$x = \frac{16}{3} = 5.3$

$3x = \frac{1}{\quad}$

Change-of-Base Formula

If a, b, and c are positive numbers with $b \neq 1$ and $c \neq 1$, then:

$$\log_c a = \frac{\log_b a}{\log_b c}$$

In particular, $\log_c a = \frac{\log a}{\log c}$ and $\log_c a = \frac{\ln a}{\ln c}$.

Evaluate: $\log_5 20$

$\frac{\log 20}{\log 5} = 1.86$

Inverses:
Switch x and y.
Isolate the log or exponent term.
Write in other format. (log $\Leftrightarrow$ exponential)
Solve for y.

Find the inverse of $y = \ln(x-4)+2$

$x = \ln(y-4)+2$

$x-2 = \ln(y-4)$

$e^{x-2} = e^{\ln(y-4)}$

$e^{x-2} = y-4$

$+4 \quad +4$

$e^{x-2}+4 = y$

Find the inverse of $y = 2^x+3$

$x = 2^y+3$

$x-3 = 2^y$

$\log_2(x-3) = \log_2 2^y$

$\log_2(x-3) = y$

Write an exponential equation $y = ab^x$ whose graph passes through (1, 12) and (3, 108).

$12 = ab^1$

$\frac{12}{b} = a$

$\frac{12}{3} = 4 = a$

$y = 4 \cdot 3^x$

$108 = ab^3$

$108 = \frac{12}{b} \cdot b^3$

$108 = 12b^2$

$9 = b^2$

$3 = b$

Write a power function $y = ax^b$ whose graph passes through (3, 4) and (6, 15).

$4 = a(3)^b$

$\frac{4}{3^b}$

$\frac{4}{3^{1.91}} = a = 0.49$

$\boxed{y = 0.49x^{1.91}}$

$15 = a(6)^b$

$15 = \frac{4}{3^b} \cdot 6^b$

$15 = 4 \cdot \left(\frac{6}{3}\right)^b$

$15 = 4 \cdot 2^b$

$\frac{15}{4} = 2^b$

$3.75 = 2^b$

$\log_2 3.75 = \log_2 2^b$

$\frac{\log 3.75}{\log 2} = b$

$1.91 = b$

Growth rate: 4%
Start 150

$y = 150(1+r)^x$

$y = 150(1.04)^x$

4% decrease

$y = 150(0.96)^x$

(4.) $y = \log_4 2x$

D: $x > 0$

R: Real

(17.) $\log_4 3x + 5\log_4 x$

$\log_4 3x + \log_4 x^5$

$\log_4 (3x)(x^5)$

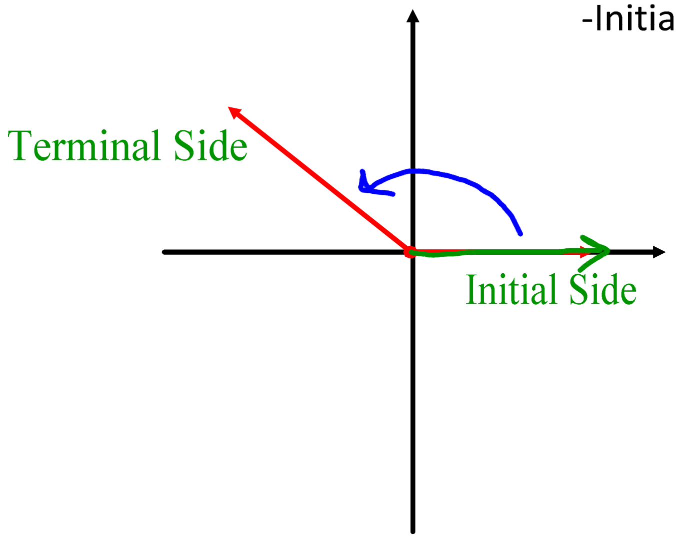
$\log_4 (3x^6)$

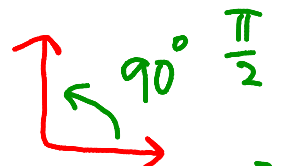
(23) $\log_3 x + \log_3(x+6) = 3$

change this

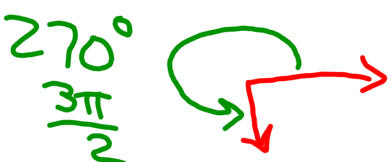
Angles in Standard Position

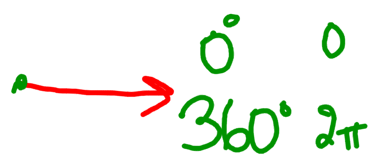
-Vertex at origin
-Initial side on positive x-axis

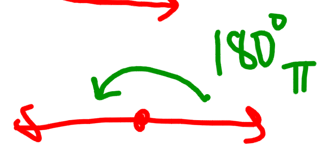




Quadrantal Angles - Terminal side is on an axis

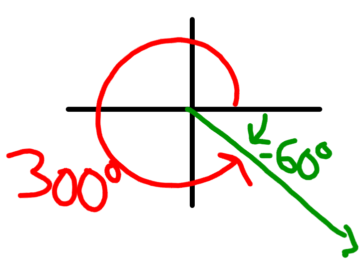


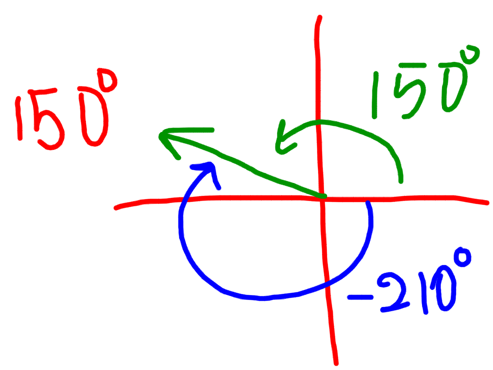


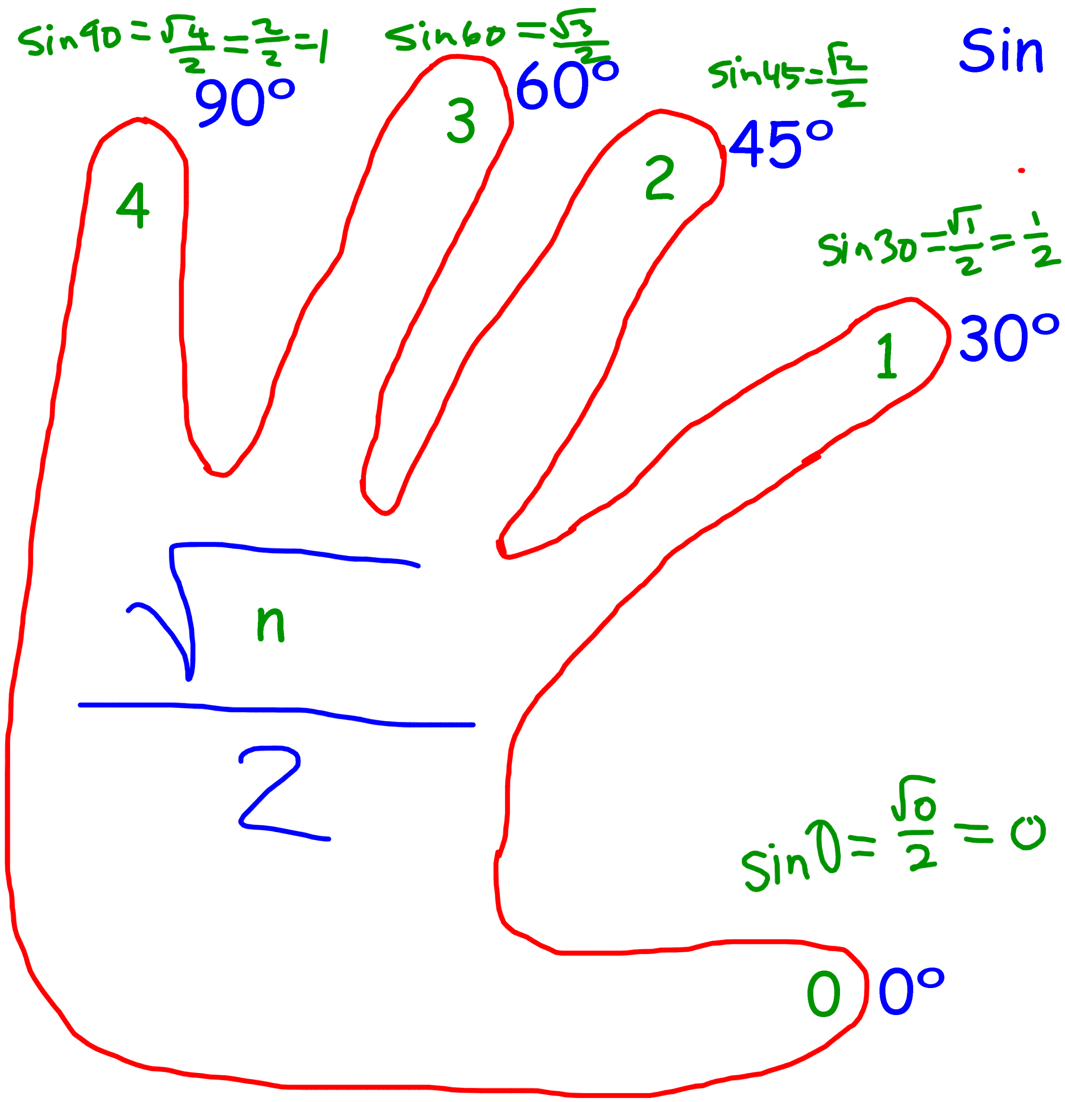


Coterminal Angles - Terminal sides coincide

Example: -60^o and 300^o







Sin
sin90 = √4/2 = 2/2 = 1
90°
4
sin60 = √3/2
60°
3
sin45 = √2/2
45°
2
sin30 = √1/2 = 1/2
30°
1
√n/2
sin0 = √0/2 = 0
0
0°

The Morman Family Trigonometry Book

Cover

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$\tan = \frac{\sin}{\cos}$

$\sin = \frac{opp}{hyp}$ $\quad \cos = \frac{adj}{hyp}$ $\quad \tan = \frac{opp}{adj}$

$\csc = \frac{1}{\sin} = \frac{hyp}{opp}$ $\quad \sec = \frac{1}{\cos} = \frac{hyp}{adj}$ $\quad \cot = \frac{1}{\tan} = \frac{adj}{opp}$

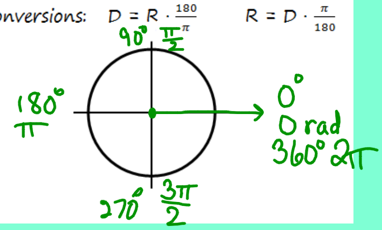
Page 1

Copy the ratios.

Radians

$180^\circ = \pi$ radians

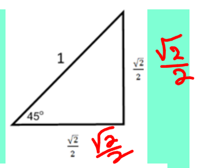
Conversions: $D = R \cdot \frac{180}{\pi}$ $\quad R = D \cdot \frac{\pi}{180}$

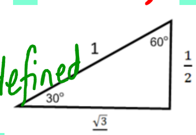


Page 2

Label each ? with radians and degrees.

Deg.	Rad.	Sin	Cos	tan
0	0	0	1	$\frac{0}{1}=0$
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90	$\frac{\pi}{2}$	1	0	$\frac{1}{0}$ undefined





$\tan 30 = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}}$

$\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\cancel{\frac{\sqrt{3}}{3} \cdot \frac{2}{1}}$

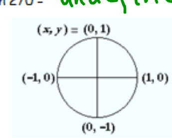
sin 0 = 0 $\quad$ cos 0 = 1 $\quad$ tan 0 = 0

sin 90 = 1 $\quad$ cos 90 = 0 $\quad$ tan 90 = $\frac{1}{0}$ = undefined

sin 180 = 0 $\quad$ cos 180 = −1 $\quad$ tan 180 = 0

sin 270 = −1 $\quad$ cos 270 = 0 $\quad$ tan 270 = undefined

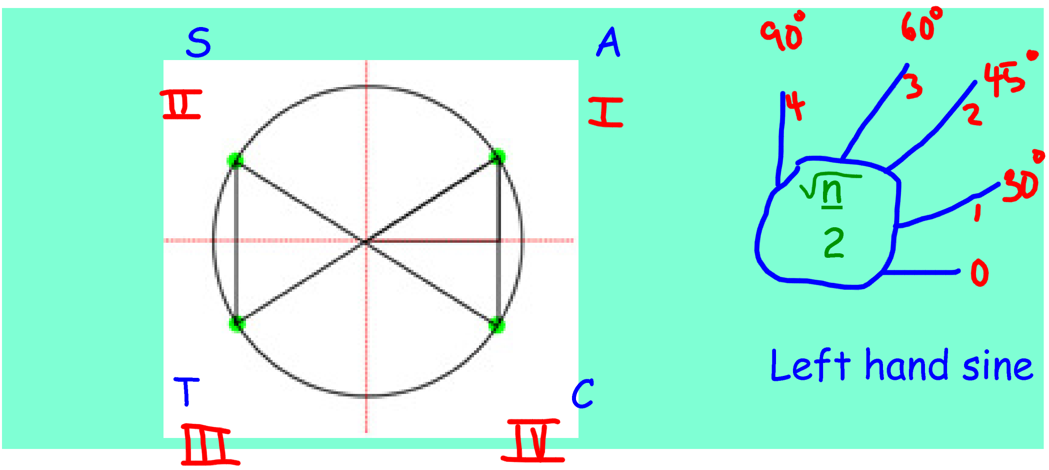
Quadrantal Angles



$(x,y) = (\cos, \sin)$

"y is sin"

$\frac{\sin}{\cos} = \tan$



Reference Angles

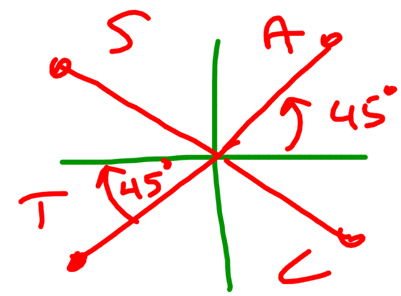
1. Determine in which quadrant the angle θ lies.
2. Determine the reference angle θ'.
3. Find the indicated ratio for θ'. This must be an exact value.
4. Determine the value for the original expression using the ASTC mneumonic.

Angle to the nearest x-axis.

$\tan \theta = 1$ what is θ ?

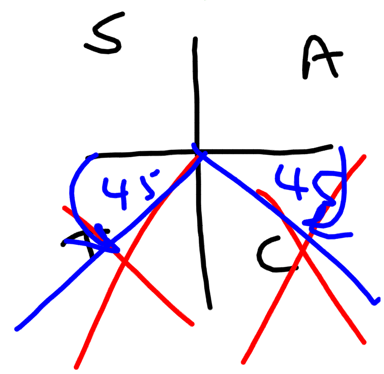
45°

225°



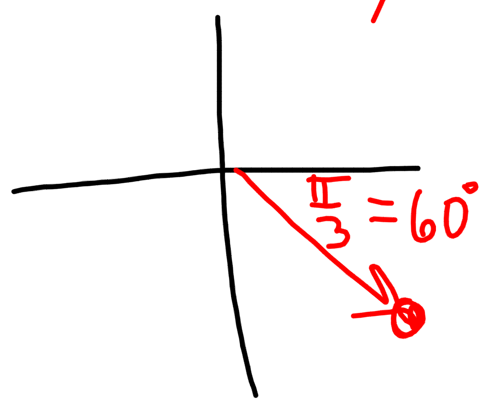
$\sin^{-1} \theta = \frac{-\sqrt{2}}{2}$ what is θ ?

225° & 315°



Find sin $\frac{5\pi}{3}$

$\frac{6\pi}{3}$



$-\frac{\sqrt{3}}{2}$

The Law of Sines

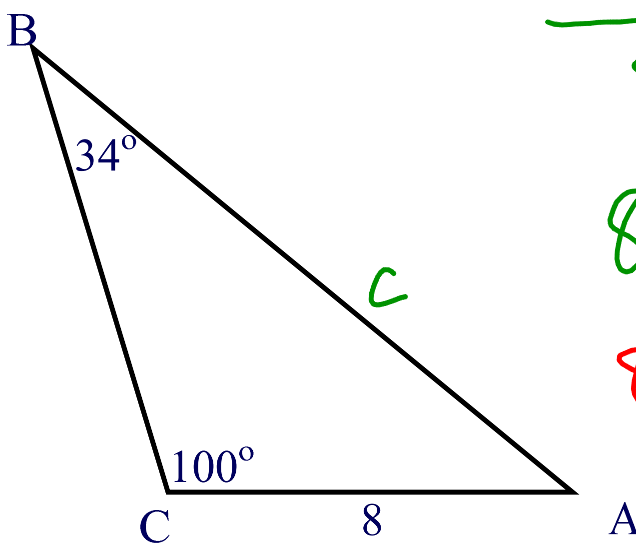
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

The Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

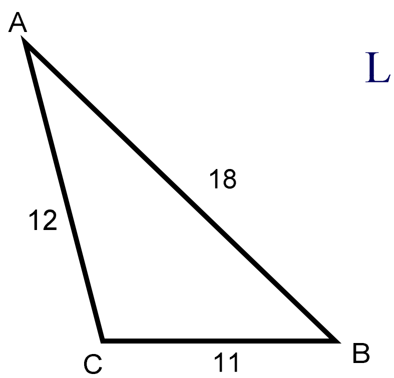


$$\frac{\sin 34}{8} = \frac{\sin 100}{c}$$

$$8\sin 100 = c\sin 34$$

$$\frac{8\sin 100}{\sin 34} \quad c$$

$$c = 14.09$$



Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos C$

Always find the biggest angle first!

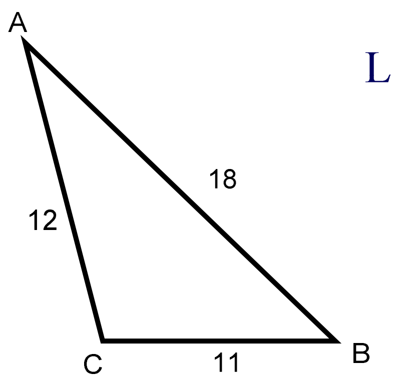
$$18^2 = 11^2 + 12^2 - 2(11)(12)\cos C$$

$$324 = 121 + 144 - 264\cos C$$

$$324 = 265 - 264\cos C$$

$$59 = -264\cos C$$

$$C = 102.9^\circ$$



Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos C$

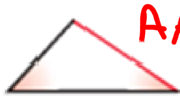
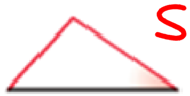
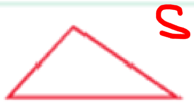
Always find the biggest angle first!

$$18^2 = 11^2 + 12^2 - 2(11)(12)\cos C$$

$$324 = 121 + 144 - 264\cos C$$

$$324 = 265 - 264\cos C$$

$$59 = -264\cos C$$

If you know this information ...		use this law ...
angle-angle-side	 AAS	Law of sines
angle-side-angle	 ASA	Law of sines
side-side-angle	 SSA	Law of sines
side-angle-side	 SAS	Law of cosines
side-side-side	 SSS	Law of cosines

SOH-CAH-TOA

Law of Sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines:

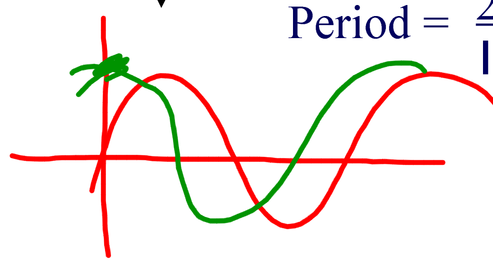
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

$y = a \sin bx$ as $a \uparrow$ amplitude $\uparrow$

$y = a \cos bx$ as $b \uparrow$ period $\downarrow$

Amplitude = $|a|$

Period = $\dfrac{2\pi}{|b|}$



sin shore

cos cliff

Graphing key points:

- max
- min
- x-intercepts

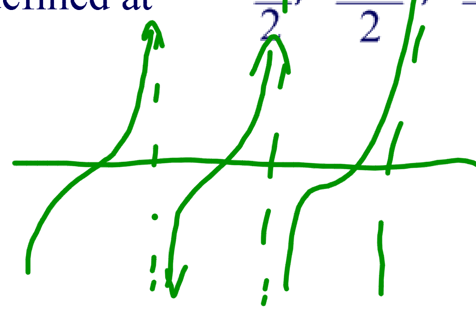
x-intercepts for $y = \sin x$ $\quad x = 0, \pi, 2\pi, 3\pi, \ldots$

x-intercepts for $y = \cos x$ $\quad x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}, \dfrac{5\pi}{2}$

$y = a \tan bx$

Tangent: undefined when cos = 0, so undefined at $x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}, \dfrac{5\pi}{2}$

No max or min $\quad$ period = $\dfrac{\pi}{|b|}$



Parent graph

$y = a \sin bx$ $\qquad$ $y = a \cos bx$

$y = a \sin b(x - h) + k$

shift parent graph horizontally h units and vertically k units
if a is negative, then reflect the graph over the midline

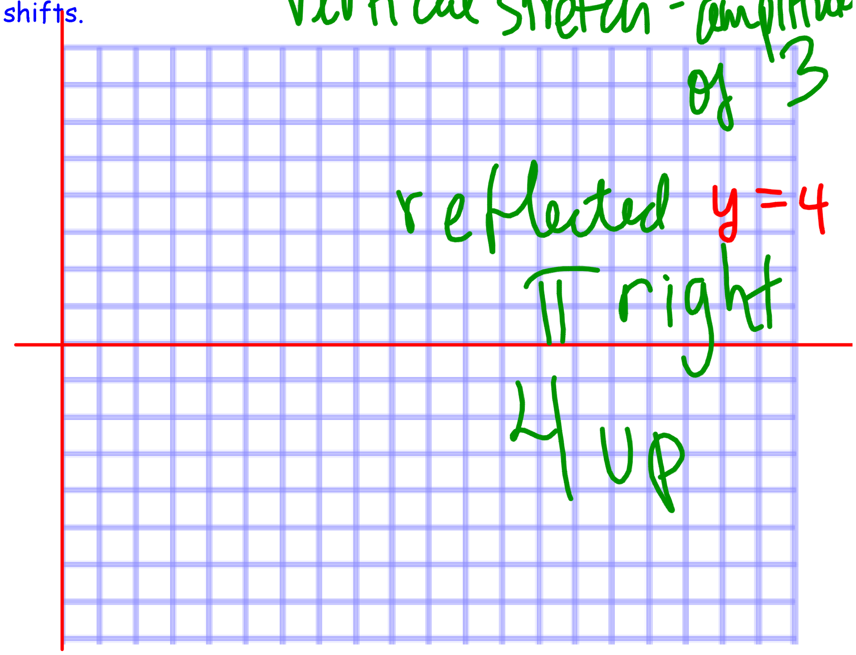
28. $y = -3\cos(x - \pi) + 4$

1. Identify the period and amplitude.
2. Number the axes.
3. Plot the parent graph: $y = 3\cos x$
4. Apply the horizontal and vertical shifts.
5. Reflect in the midline ($y = 4$)

$y = \cos x$

period $= \frac{2\pi}{b} = 2\pi$

vertical stretch - amplitude of 3



reflected $y=4$

π right

4 up

Direct Variation: $y = ax$

The total cost for a pizza party varies directly with the number of pizzas bought.

$\frac{y}{x} = 5$

$y = 5x$

y = total cost, x = # pizzas ($5/pizza)

Inverse Variation: $y = \frac{a}{x}$

$xy = a$

Constant of Variation

5 pizzas (40 pieces) are shared by a group. The amount of pizza per person varies inversely with the number of people in the group.

$y = \frac{40}{x}$

y = slices per person x = # people (40 slices total)

The distance between Chicago and Minneapolis is about 400 miles.

The drive time (y) in hours, varies inversely with the speed (x) in miles per hour.

$x = 10$ $\quad y = \frac{400}{10} = 40$ hours

$x = 20$ $\quad y = \frac{400}{20} = 20$ hours

$x = 60$ $\quad y = \frac{400}{60} = 6\frac{2}{3}$ hours

$x = 70$ $\quad y = \frac{400}{70} = 5.71$ hours

$$f(x) = \frac{ax^m + \ldots}{bx^n + \ldots}$$

$$\frac{3x^4 - 5x^2 + 3x}{2x^5 - 4x^4 + 3x^2 \ldots}$$

x-intercepts: Zeros of the numerator

Vertical Asymptotes: Zeros of the denominator

Horizontal Asymptotes:

if $m < n$ then $y = 0$ is a HA

if $m = n$ then $y = \frac{a}{b}$ is a HA

if $m > n$, then there is no HA

Plot several points on either side of each vertical asymptote.

$$\frac{80x^4}{y^3} \cdot \frac{xy}{5x^2} = \frac{\cancel{80}^{16} x^{\cancel{5}^{3}} \cancel{y}}{\cancel{5}_{1} \cancel{x^2} y^{\cancel{3}\,2}} = \boxed{\frac{16x^3}{y^2}}$$

$$\frac{x^2 - 13x + 40}{x^2 - 2x - 15} \div \frac{(x^2 - 5x - 24)}{1} = \frac{\cancel{(x-8)}\cancel{(x-5)}}{\cancel{(x-5)}(x+3)} \cdot \frac{1}{\cancel{(x-8)}(x+3)}$$

$$= \frac{1}{(x+3)^2}$$

$$\frac{5}{6(x+3)} + \frac{x+4}{2x}$$

$6x(x+3)$

$$\frac{5}{6(x+3)} \cdot \frac{x}{x} + \frac{(x+4)}{2x} \cdot \frac{3(x+3)}{3(x+3)}$$

$$\frac{5x + 3(x^2+7x+12)}{6x(x+3)}$$

$$\frac{5x + 3x^2 + 21x + 36}{6x(x+3)} = \frac{3x^2+26x+36}{6x(x+3)}$$

$$\frac{5}{x} = \frac{7}{x+2}$$

$7x = 5(x+2)$

$(x+1)(x-1)$

$$\left[\frac{3x}{x+1} = \frac{12}{x^2-1} + 2\right] = \left[\frac{3x}{x+1} = \frac{12}{(x+1)(x-1)} + 2\right]$$

$3x(x-1) = 12 + 2(x+1)(x-1)$

$3x^2 - 3x = 12 + 2x^2 - 2$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x = 5, -2$

$$\frac{\frac{x}{3} + 2}{\frac{1}{x} + 3} \cdot \frac{3x}{3x}$$

$$\boxed{\frac{x^2+6x}{3+9x}}$$

$$\boxed{\frac{x(x+6)}{3(1+3x)}}$$

27) $$\frac{x}{x^2-9}+\frac{x+1}{x^2+6x+9}$$

$$\frac{x}{(x+3)(x-3)}+\frac{x+1}{(x+3)(x+3)}$$

$$\frac{x}{(x+3)(x-3)}\cdot\frac{(x+3)}{(x+3)}=\boxed{\frac{x^2+3x}{(x+3)(x+3)(x-3)}}$$

$$\frac{(x+1)}{(x+3)(x+3)}\cdot\frac{(x-3)}{(x-3)}=\boxed{\frac{x^2-2x-3}{(x+3)(x+3)(x-3)}}$$

$$\frac{x^2+3x+x^2-2x-3}{(x+3)(x+3)(x-3)}=\frac{2x^2+x-3}{(x+3)(x+3)(x-3)}$$

$$2x(x+1)\left[\frac{3x}{x+1} - \frac{5}{2x} = \frac{3}{2x}\right]$$

$$6x^2 - 5(x+1) = 3(x+1)$$

$$6x^2 - 5x - 5 = 3x + 3$$

$$6x^2 - 8x - 8 = 0$$

$$2(3x^2 - 4x - 4) = 0$$

$$2(3x+2)(x-2) = 0$$

$3x + 2 = 0$ $\qquad$ $x - 2 = 0$

$3x = -2$ $\qquad$ $x = 2$

$$x = \frac{-2}{3}$$

$$\frac{5x}{x-2} = 7 + \frac{10}{x-2}$$

Distance Formula: $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Midpoint Formula: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Permutations of *n* Objects Taken *r* at a Time

The number of permutations of *r* objects taken from a group of *n* distinct objects is denoted by ${}_nP_r$ and is given by this formula:

$${}_nP_r = \frac{n!}{(n-r)!}$$

The number of ways 3 objects can be arranged from a group of 5 objects.

$${}_5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!}$$

A B C D E

(ABC is different from BCA)

19 - 3 - 21

3 - 19 - 21

Permutations with Repetition

The number of distinguishable permutations of n objects where one object is repeated s_1 times, another is repeated s_2 times, and so on, is:

$$\frac{n!}{s_1! \cdot s_2! \cdot \ldots \cdot s_k!}$$

Find the number of distinguishable permutations of the letters in

SWIMMING $= \dfrac{8!}{2! \cdot 2!} =$

Combinations of n Objects Taken r at a Time

The number of combinations of r objects taken from a group of n distinct objects is denoted by ${}_nC_r$ and is given by this formula:

$${}_nC_r = \frac{n!}{(n-r)! \cdot r!}$$

5-card hands dealt from a deck of 52 cards:

$$\frac{52!}{(52-5)\cdot 5!}$$

KEY CONCEPT *For Your Notebook*

Measures of Central Tendency

- The **mean**, or *average*, of n numbers is the sum of the numbers divided by n. The mean is denoted by $\bar{x}$, which is read as "x-bar." For the data set $x_1, x_2, \ldots, x_n$, the mean is $\bar{x} = \dfrac{x_1 + x_2 + \cdots + x_n}{n}$.
- The **median** of n numbers is the middle number when the numbers are written in order. (If n is even, the median is the mean of the two middle numbers.)
- The **mode** of n numbers is the number or numbers that occur most frequently. There may be one mode, no mode, or more than one mode.

Measures of Dispersion A *measure of dispersion* is a statistic that tells you how dispersed, or spread out, data values are. One simple measure of dispersion is the **range,** which is the difference between the greatest and least data values.

Standard Deviation Another measure of dispersion is *standard deviation*, which describes the typical difference (or *deviation)* between a data value and the mean.

Standard Deviation of a Data Set

The **standard deviation** σ (read as "sigma") of $x_1, x_2, \ldots, x_n$ is:

$$\sigma = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}}$$

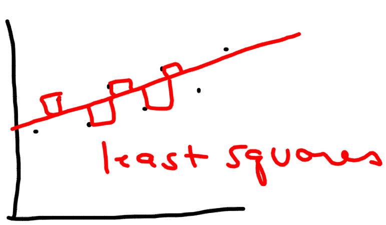
$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

1. Find the mean ($\bar{x}$)
2. Find the difference between each data value and $\bar{x}$ then square the difference.
3. Add the squared differences (Σ)
4. Divide by the number of data points (n)
5. Take the square root. The answer is σ, the standard deviation.

7.) 12, 8, 17, 15, 12, 14

Range = 17 − 8 = 9 mean ($\bar{x}$) = 13

$1^2 + 5^2 + 4^2 + 2^2 + 1^2 + 1^2$

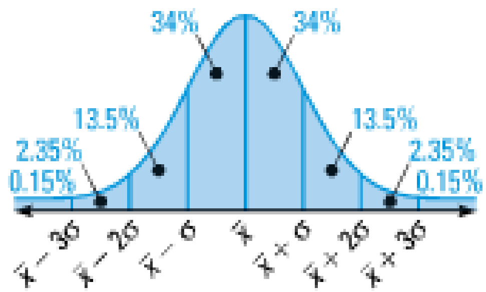


$\frac{48}{6} = 8$

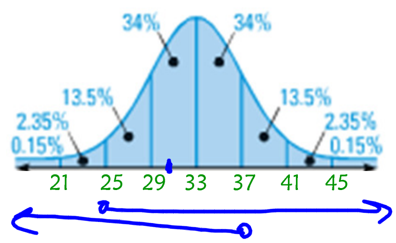
$\sqrt{8} \approx 2.8$

A normal curve is defined by an equation of this form:

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\bar{x}}{\sigma}\right)^2}$$



A normal distribution has a mean of 33 and a standard deviation of 4.



Find the probability that a randomly selected x-value from the distribution is in the given interval.

Between 29 and 37 68%

At least 25 $100\% - (2.35 + .15) = 97.5\%$

At most 37

30 or less: **Formula:** $z = \frac{x - \bar{x}}{\sigma}$

$$\frac{30-33}{4} = -\frac{3}{4} = -0.75$$

$$= -0.8$$

$P(x \le 30) = 0.2119$

.2119

STANDARD NORMAL TABLE If z is a randomly selected value from a standard normal distribution, you can use the table below to find the probability that z is less than or equal to some given value. For example, the table shows that $P(z \le -0.4) = 0.3446$. You can find the value of $P(z \le -0.4)$ in the table by finding the value where row −0 and column .4 intersect.

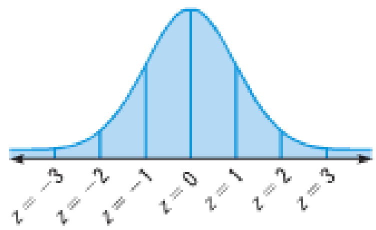
Standard Normal Table

z	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
−3	.0013	.0010	.0007	.0005	.0003	.0002	.0002	.0001	.0001	.0000+
−2	.0228	.0179	.0139	.0107	.0082	.0062	.0047	.0035	.0026	.0019
−1	.1587	.1357	.1151	.0968	.0808	.0668	.0548	.0446	.0359	.0287
−0	.5000	.4602	.4207	.3821	.3446	.3085	.2743	.2420	.2119	.1841
0	.5000	.5398	.5793	.6179	.6554	.6915	.7257	.7580	.7881	.8159
1	.8413	.8643	.8849	.9032	.9192	.9332	.9452	.9554	.9641	.9713
2	.9772	.9821	.9861	.9893	.9918	.9938	.9953	.9965	.9974	.9981
3	.9987	.9990	.9993	.9995	.9997	.9998	.9998	.9999	.9999	1.0000−

STANDARD NORMAL DISTRIBUTION The **standard normal distribution** is the normal distribution with mean 0 and standard deviation 1. The formula below can be used to transform x-values from a normal distribution with mean $\overline{x}$ and standard deviation σ into z-values having a standard normal distribution.

Formula: $z = \dfrac{x - \overline{x}}{\sigma}$

Subtract the mean from the given x-value, then divide by the standard deviation.



The z-value for a particular x-value is called the **z-score** for the x-value and is the number of standard deviations the x-value lies above or below the mean $\overline{x}$.

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−3	.0013	.0010	.0007	.0005	.0003	.0002	.0002	.0001	.0001	.0000+
−2	.0228	.0179	.0139	.0107	.0082	.0062	.0047	.0035	.0026	.0019
−1	.1587	.1357	.1151	.0968	.0808	.0668	.0548	.0446	.0359	.0287
−0	.5000	.4602	.4207	.3821	.3446	.3085	.2743	.2420	.2119	.1841
0	.5000	.5398	.5793	.6179	.6554	.6915	.7257	.7580	.7881	.8159
1	.8413	.8643	.8849	.9032	.9192	.9332	.9452	.9554	.9641	.9713
2	.9772	.9821	.9861	.9893	.9918	.9938	.9953	.9965	.9974	.9981
3	.9987	.9990	.9993	.9995	.9997	.9998	.9998	.9999	.9999	1.0000−

BIOLOGY Scientists conducted aerial surveys of a seal sanctuary and recorded the number x of seals they observed during each survey. The numbers of seals observed were normally distributed with a mean of 73 seals and a standard deviation of 14.1 seals. Find the probability that at most 50 seals were observed during a survey.

Solution

STEP 1 Find the z-score corresponding to an x-value of 50.

$$z = \frac{x - \overline{x}}{\sigma} = \frac{50 - 73}{14.1} \approx -1.6$$

STEP 2 Use the table to find $P(x \le 50) \approx P(z \le -1.6)$.

The table shows that $P(z \le -1.6) = 0.0548$. So, the probability that at most 50 seals were observed during a survey is about 0.0548.

WHAT IF? In Example 3, find the probability that at most 90 seals were observed during a survey.

Find the probability that between 60 and 70 seals were observed.